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**Differences in the Ability of Structural and Reduced-
Form Models to Improve Pricing Decisions**

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Abstract

Although structural modeling as a means to improve pricing decisions often appears in prestigious academic journals, its consideration in pricing textbooks remains rather limited, and knowledge about its benefits and limitations in comparison to reduced-form models is scarce among "structural nonexperts". This article outlines for these "structural nonexperts" which different abilities structural and reduced-form models have to improve pricing decisions. Therefore, it uses a popular textbook example that fails to consider the abilities of both models, which are the abilities to capture the effects of consumers' responses and changes in cost structure on prices. It outlines that the major advantages of structural models are the ability to capture competitors' reactions and to describe new market equilibriums, which come at the costs of assumptions that usually prevent structural models to further improve prices. The beauty of the simple example provided is that it enables nontechnical readers to easily understand the major differences between structural and reduced-form models.

1 Introduction

Textbooks (e.g., Simon/Fassnacht (2009); Wübker (2006)) and consulting companies (e.g., Simon-Kucher & Partners, Noecker and Company) frequently illustrate the importance of pricing with the example described in Table 1, according to which a firm experiences variable costs of 60€ per unit and fixed costs of 30€ million and charges a price of 100€ (Marn/Roegner/Zawada (2004) and Baker (2006) use comparable examples.) The firm thus realizes revenues of 100€ million, a profit contribution of 40€ million, and a total profit of 10€million.

Table 1: Textbook example of the importance of pricing

Driver	Old Value	New Value	Old Profit	New Profit	Change in Profit
Price	100€	110€	10,000,000€	20,000,000€	+100%
Variable costs	60€	54€	10,000,000€	16,000,000€	+60%
Quantity	1,000,000	1,100,000	10,000,000€	14,000,000€	+40%
Fixed costs	30,000,000€	27,000,000€	10,000,000€	13,000,000€	+30%

To highlight the importance of the price, users of this example illustrate the effects of a 10% improvement in price, quantity, or costs. The 10% increase in price leads to an increase in profit of 100%, which is much greater than the other effects. The crucial assumption in this example obviously is that changes in price, costs, or quantity have no effect on one another, even though such independence is unlikely in the real world. Quantity shifts in response to changes in price, price depends on changes in cost, and the prices of the firm's competitors reflect changes in the firm's own price.

The basic idea of structural modeling is to capture all of these effects, while reduced-form modeling only captures some of them. Frequently, structural models result in fairly technical papers that are rather difficult for nonexperts in the field—referred to herein as "structural nonexperts"—to understand. This article therefore attempts to outline for these structural non-

experts which different abilities structural and reduced-form models have to improve pricing decisions. By extending the simple textbook example in Table 1, this effort demonstrates how structural modeling can capture the effects of consumers' responses, competitors' reactions, and changes in cost structure and how these models differ from reduced-form models. It also outlines the assumptions required for structural modeling.

Section 2 describes the basic idea of structural and reduced-form models, followed by Sections 3–5, which address the three components required for structural models, namely, the effects of consumers' responses, cost changes, and competitors' responses, respectively, on price changes. The first two components are usually also considered in reduced-form models. Section 6 outlines the influence of the functional form of the price response model on price changes in structural models, and Section 7 summarizes the main results and conclusions.

2 Basic Idea of Structural and Reduced-Form Models

Structural models became popular with the rise of research into the "New Empirical Industrial Organization" (NEIO) (Kadiyali/Sudhir/Rao (2001)). They attempt to model the impact of the firm's strategic marketing mix decisions on its profit by capturing the initial impact of the decisions on (1) consumer demand for the firm's products, (2) competitors' strategic marketing mix decisions, and (3) the subsequent impact of consumers' demand on the firm's costs (Kadiyali/Sudhir/Rao (2001); Leeflang (2008)). A counterpart of a structural model is a reduced-form model, sometimes denominated rather colloquially as the "data-driven approach" (Chintagunta/Erdem/Rossi/Wedel (2006)). The major difference between these models is that structural models explicitly capture the optimal strategic behavior of firms and sometimes that of consumers (Chan/Kadiyali/Xiao (2009)). That is, consumers tend to maximize their utility, usually through nonstrategic behaviors. In this context, nonstrategic behavior means that (1) one consumer's choice has no impact on another consumers' choices, so their choices all can be assumed to be independent (Kadiyali/Sudhir/Rao (2001)), and (2) consumers do not take

the firms' reaction to their behavior into account, which means that they do not try to hide their true price sensitivity in an effort to encourage firms to charge different prices. In contrast, firms behave strategically, because their consumers and competitors react to their marketing mix decisions.

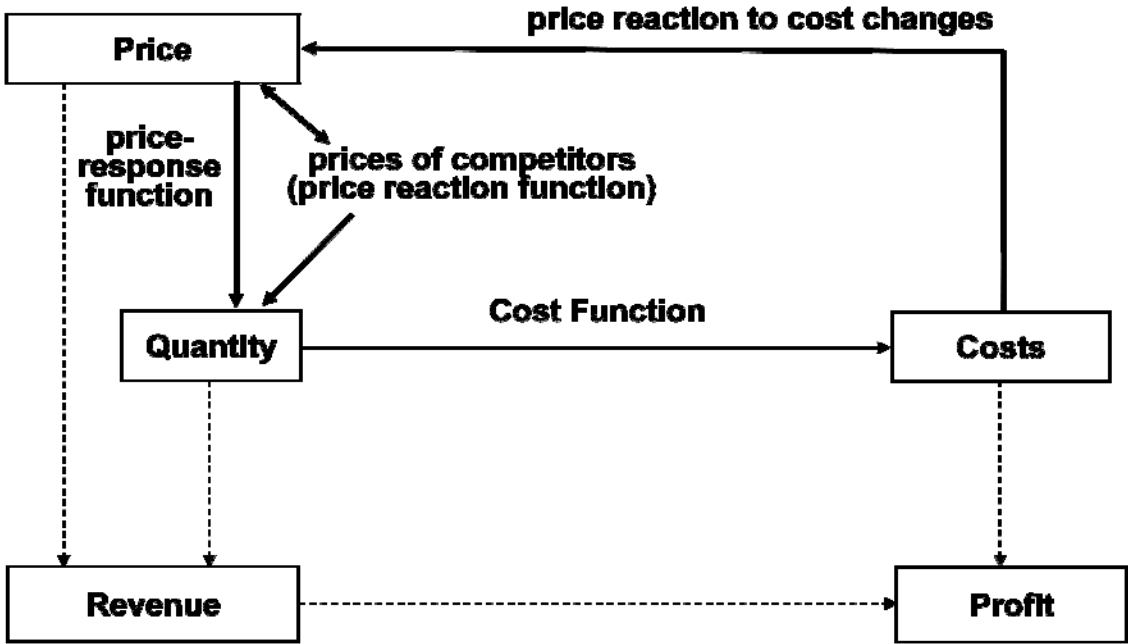
Modeling optimal behavior, including the optimal reactions of competitors, requires certain behavioral assumptions, and theories should support such assumptions (Chan/Kadiyali/Xiao (2009)). Structural models thus frequently are described as relying "on economic and/or marketing theories of consumer or firm behavior to derive the econometric specification that can be taken to data" (Chintagunta/Erdem/Rossi/Wedel (2006), p. 604). I find this denomination misleading though; most researchers likely agree that models that rely on theories are better than models that rely just on data. Yet both structural and (nonstructural) reduced-form models depend on theories (e.g., that price influences demand), and the use or nonuse of theories does not create their major differences. Instead, their divergences result from the assumptions the models make about the strategic behavior of firms and whether this strategic behavior can be tested by comparing the fit of models with varying assumptions about the firms' strategic behaviors. For example, Kadiyali/Vilcassim/Chintagunta (1996) do so by deriving the first-order conditions for the optimal prices under different pricing strategies (and the corresponding market equilibriums) and then choose the pricing strategy that fits the data best. As a result, Kadiyali/Vilcassim/Chintagunta (1996) conclude that their considered firms likely engage in Stackelberg leader-follower pricing competition (for a comparable approach see Richards (2007); Roy/Hanssens/Raju (1994)). While I agree that this approach is certainly a very appealing form to test for different kind of competitive strategies, I also feel that this approach is very "data-driven".

In their basic form, structural models assume that firms behave optimally. Therefore, they explain observed behavior as a result of optimization, which defines a market equilibrium

(Chintagunta/Erdem/Rossi/Wedel (2006)). Firms' marketing mix decisions cannot be improved, because every firm already behaves optimally. But such a result is unappealing for normative researchers who want to provide guidance to firms and therefore prefer reduced-form models. Reduced-form models usually can outline some potential improvements in pricing decisions that are not possible to derive from structural models. However, structural models provide guidance if market environments change; for example, they can describe the new market equilibrium that results if the product cost increases due to higher taxes or increased commodity prices. Such an equilibrium analysis is not possible with reduced-form models (Chintagunta/Erdem/Rossi/Wedel (2006)).

Figure 1 describes the drivers and effects of prices, building on the example from Table 1. For the sake of simplicity, I consider price as the only marketing mix instrument. A firm's price influences both the quantity and the price of the competitor (thick lines in Figure 1). The quantity influences the costs, perhaps due to scale or learning effects, but for the sake of simplicity, I assume that fixed costs and variable costs per unit are constant and thus not influenced by quantity. Changes in costs influence prices though (thick line), and obviously, price, costs, and quantity determine revenues and profits. This model is much simpler than are most published structural models (for an overview of more elaborate models, see Reiss/Wolak (2007); Chan/Kadiyali/Xiao (2009)), but it still captures the strategic behavior of firms and offers an understanding of the basic idea of structural models. It is admittedly somewhat deficient in terms of capturing the optimal consumer behavior, because it captures this behavior only with a simple (linear or multiplicative) price-response function. A more elaborate structural model would use a logit model to mimic the behavior of utility maximizing consumers (e.g., Draganska/Klapper/Villas-Boas (2009); Klapper (2003)).

Figure 1: Drivers and effects of prices



In the next three sections, I describe how to capture the consumers' response to changes in the firm's price (i.e., price response function), the price reaction to cost changes, and the reaction of competitors' prices to changes in the focal firm's prices. All three components are required in structural models. Reduced-form models instead usually capture the consumers' responses to changes in the prices of the firm, and sometimes changes in costs, but do not consider competitors' reactions.

3 Capturing the Effects of Consumers' Responses on Price Changes

Consumers generally react to changes in prices, and researchers typically measure these reactions according to price elasticities. Tellis (1988) finds an average price elasticity of -1.76 across the 367 price elasticities published in 42 studies during 1962–1985. Bijmolt/van Heerde/Pieters (2005) report an average price elasticity of -2.62, based on 1,851 price elasticities from 81 studies covering the years 1961 to 2004. Price response functions can capture consumer reactions to price and measure price elasticities. I start by assuming a linear price response function, which is fairly popular in structural models (Klapper (2003)):

(1) $q = a + b \cdot p$

where:

q: quantity,
p: price, and
a, b: parameters of the linear price response function.

The elasticity in the linear price response function is $\varepsilon^{linear} = \frac{b \cdot p}{q} = \frac{b \cdot p}{a + b \cdot p}$. Assuming a price elasticity of -2 for the new price of 110€ and sales of 1 million units, the price response function that corresponds to the numerical example in Table 1 is:

$$(2) \quad q(p) = 2,818,180 - 18,182 \cdot p,$$

and the corresponding profit at the new price of 110€ would be $\Pi = 10,909,000\text{€}$ ¹

The optimal price then can be derived easily by optimizing the following profit function:

$$(3) \quad \pi = (p - c) \cdot q(p) - C_{fix},$$

where:

c: variable costs, and
 C_{fix} : fixed costs.

The first-order conditions yield the optimal price:

$$(4) \quad p = \frac{b \cdot c - a}{2 \cdot b},$$

As I show in Table 2, the optimal price is $p = 107.50\text{€}$, which leads to a profit of $\Pi = 11,022,734\text{€}$. Not surprisingly, this profit is substantially lower than the 20€million profit in the textbook example in Table 1—a clear demonstration of the shortcoming of this example.

¹ The profit would be exactly 10,000,000€ if I were to interpret the price elasticity of -2 as an arc elasticity (and not a point elasticity). In this case, the quantity would drop by 20% to 800,000 and result in a profit contribution of $50\text{€} \cdot 800,000 = 40,000,000\text{€}$ and a profit of 10€million.

Table 2: Summary of results that capture price responses (linear price response function)

	Current Situation	Price Increase (without consumer reaction)	Price Increase (with consumer reaction)	Optimal Price
Price	100.00€	110.00€	110.00€	107.50€
Quantity	1,000,000	1,000,000	818,182	863,636
Profit contribution	40,000,000€	50,000,000€	40,909,100€	41,022,734€
Profit	10,000,000€	20,000,000€	10,909,000€	11,022,734€
Notes: Variable costs = 60€, fixed costs = 30,000,000€				

4 Capturing the Effects of Cost Changes

Equation (4) reveals that the optimal price depend on the consumers' response (as measured by the parameters of the price response function) and the variable costs c . There is no direct effect of changes in the costs on consumers' responses, but there is an indirect effect, because firms adjust their prices if their variable costs change (see Equation (4)). To outline those effects, I start with the optimal price that I just derived (Table 2). If firms do not adjust their prices, the results in Table 3 show that a decrease in the variable cost by 10% leads to an increase in profit, from 11,022,734€ to 16,204,550€

Table 3: Summary of results that capture reactions to changes in costs (linear price response function)

	Optimal Price (without cost decrease)	Cost Decrease (without price adjustment)	Optimal Price (in case of cost decrease)
Price	107.50€	107.50€	104.50€
Variable cost per unit	60.00€	54.00€	54.00€
Quantity	863,636	863,636	918,181
Profit contribution	14,022,734€	46,204,550€	46,368,186€
Profit	11,022,734€	16,204,550€	16,368,186€
Notes: Variable costs = 60€, fixed costs = 30,000,000€			

Firms can further increase their profits by lowering prices too. Thus, if a firm uses Equation (4) to determine its optimal prices, it should use a cost decrease to lower its price from 107.50€ to 104.50€, which increases its profit from 16,204,550€ to 16,368,186€

5 Capturing the Effects of Competitors' Reactions on Price Changes

Thus far, I have assumed a monopoly for the focal firm, though most firms function in competitive settings, which means they must also consider competitors' reactions. For ease of exposition in this section, I assume that the firm has just one competitor ($i = 2$). In this case, the linear response function in Equation (1) can be extended as follows (Klapper (2003); Kadiyali/Sudhir/Rao (2001)):

$$(5) \quad q_1 = a_1 + b_{11} \cdot p_1 + b_{12} \cdot p_2, \text{ and}$$

$$(6) \quad q_2 = a_2 + b_{21} \cdot p_1 + b_{22} \cdot p_2.$$

More generally, these price response functions can be described as:

$$(7) \quad q_i = a_i + b_{ii} \cdot p_i + b_{ij} \cdot p_j \quad (i = 1, 2).$$

Therefore, the profit function is:

$$(8) \quad \pi_i = (p_i - c_i) \cdot q_i - C_{fix,i} \quad (i = 1, 2).$$

Game theory now can help the firms set prices optimally. The three most common behavioral regimes are the Bertrand-Nash price competition, Stackelberg leader-follower price competition, and collusion (Kadiyali/Sudhir/Rao (2001); Chintagunta/Erdem/Rossi/Wedel (2006); Chan/Kadiyali/Xiao (2009); Klapper (2003)). For an overview of other behavioral regimes see Leeflang (2008) and the literature cited there.

5.1 Bertrand-Nash Pricing

In Bertrand competition, all firms simultaneously set their prices, and competitors cannot increase their profits by changing their prices. In such a situation, $\frac{\partial p_j}{\partial p_i} = 0$. This price-setting scenario results in a Nash equilibrium, denoted by a pair of prices that allow neither firm to increase its profits by unilaterally changing its price. Thus, the first-order condition for firm i in the Bertrand-Nash pricing equilibrium is (Kadiyali/Sudhir/Rao (2001)):

$$(9) \quad \frac{\partial \pi_i}{\partial p_i} = (p_i - c_i) \cdot \frac{\partial q_i}{\partial p_i} + q_i = 0 \quad (i = 1, 2).$$

With a linear demand function (see Equation (7)), Equation (9) produces:

$$(10) \quad \frac{\partial \pi_i}{\partial p_i} = (p_i - c_i) \cdot b_{ii} + q_i = 0 \quad (i = 1, 2).$$

Substituting Equation (7) into Equation (10) and rearranging then yields:

$$(11) \quad p_i = \frac{b_{ii} \cdot c_i - a_i - b_{ij} \cdot p_j}{2 \cdot b_{ii}} \quad (i = 1, 2).$$

Rewriting Equation (11) for both firms separately yields the following two equations, which allow for determining the optimal prices for both firms numerically (for slightly different formulations that also allow for numerically finding the optimal prices see Kadiyali/Sudhir/Rao (2001)):

$$(12) \quad p_1 = \frac{b_{11} \cdot c_1 - a_1 - b_{12} \cdot p_2}{2 \cdot b_{11}}, \text{ and}$$

$$(13) \quad p_2 = \frac{b_{22} \cdot c_2 - a_2 - b_{21} \cdot p_1}{2 \cdot b_{22}}.$$

When $b_{12} = 0$ and $b_{21} = 0$, Equations (12) and (13) correspond to Equation (4). Inserting Equation (12) in Equation (13) and rearranging also allows for determining the optimal price for the second firm:

$$(14) \quad p_2 = \frac{b_{22} \cdot c_2 - a_2 - \frac{b_{21} \cdot c_1}{2} + \frac{b_{21} \cdot a_1}{2 \cdot b_{11}}}{2 \cdot b_{22} - \frac{b_{21} \cdot b_{12}}{2 \cdot b_{11}}}.$$

The similar procedure also determines the optimal price for the first firm:

$$(15) \quad p_1 = \frac{b_{11} \cdot c_1 - a_1 - \frac{b_{12} \cdot c_2}{2} + \frac{b_{12} \cdot a_2}{2 \cdot b_{22}}}{2 \cdot b_{11} - \frac{b_{12} \cdot b_{21}}{2 \cdot b_{22}}}.$$

Inserting these prices in Equations (5) and (6) then allows for the determination of corresponding quantities.

I again use a numerical example to illustrate the effect of capturing competitors' reaction for the optimal prices derived in Table 2. To consider the impact of the competitor's price, I arbitrarily assume $b_{21} = 5000$, with $p_2 = 100\text{€}$ and I modify the intercept in the response function in Equation (2) to retain the quantity $q_1 = 1,000,000$ for $p_1 = 100\text{€}$ and $p_2 = 100\text{€}$

$$(16) \quad q_1(p_1, p_2) = 2,318,180 - 18,182 \cdot p_1 + 5,000 \cdot p_2.$$

For the sake of simplicity, I assume that the price response function of the second firm is comparable to that of the first firm, which means that the firms are symmetric:

$$(17) \quad q_2(p_1, p_2) = 2,318,180 - 18,182 \cdot p_2 + 5,000 \cdot p_1.$$

I also assume that the second firm has the same variable and fixed costs. Table 4 displays the result of Bertrand-Nash pricing behavior. The different rounds describe the adjustment

process. In round 1, firm 1 optimizes its price (here $p_1=107.50\text{€}$) by not considering the competitor's reaction to its price, that is, when it assumes p_2 remains 100€ . This result corresponds to the result in Table 2. Yet firm 2 seems certain to react. If it can anticipate that the first firm's price is 107.50€ , it will set its price to $p_2 = 108.53\text{€}$, which is the result described in round 2. Interestingly, this adjustment in price of the second firm increases the first firm's profit even stronger than the one of the second firm. Yet, firm 1 also anticipates this behavior of firm 2 so that it will set its price to $p_1 = 108.67\text{€}$ (round 3), which increases the first firm's profit, but even stronger increases the second firm's profit. However, as firm 2 also anticipates such a behavior, it will set a different price (round 4). After two more rounds, the equilibrium prices of 108.70€ and a corresponding profit of $13,113,953\text{€}$ for both firms are reached. Thus, in a model of Bertrand-Nash pricing behavior both firms simultaneously set their price and anticipate the competitor's reaction to its price. The equilibrium describes a situation in which no firm has an incentive to deviate from its price.

Table 4: Summary of results for Bertrand-Nash Pricing (linear case)

	Current Situation	Round 1 (firm 1 optimizes its price, firm 2 does not react)	Round 2 (firm 2 optimizes its price, firm 1 does not react)	Round 3 (firm 1 optimizes its price, firm 2 does not react)	Round 4 (firm 2 optimizes its price, firm 1 does not react)	Round 5 (firm 1 optimizes its price, firm 2 does not react)	Round 6 (firm 2 optimizes its price, firm 1 does not react)	Equilibrium Prices
Price of firm 1	100.00€	107.50€	107.50€	108.67€	108.67€	108.70€	108.70€	108.70€
Price of firm 2	100.00€	100.00€	108.53€	108.53€	108.69€	108.69€	108.70€	108.70€
Quantity of firm 1	1,000,000	863,636	906,293	884,964	885,771	885,367	885,383	885,383
Profit of firm 1	10,000,000€	11,022,734€	13,048,913€	13,073,934€	13,113,188€	13,113,197€	13,113,939€	13,113,939€
Quantity of firm 2	1,000,000	1,037,500	882,386	888,251	885,319	885,430	885,374	885,374
Profit of firm 2	10,000,000€	11,500,006€	12,823,321€	13,107,977€	13,108,450 €	13,113,849€	13.113.849€	13.113.849€
Notes: Variable costs = 60€, fixed costs = 30,000,000								

5.2 Stackelberg Leader-Follower Price Competition

In a Stackelberg leader-follower price competition, one firm is the leader (here, firm 1), and the other firm is the follower (firm 2). The follower assumes that the leader does not react to changes in the price of the follower, $\frac{\partial p_1}{\partial p_2} = 0$, but the leader anticipates that the follower will

react to its price p_1 , $\frac{\partial p_2}{\partial p_1} \neq 0$. Hence, the first-order condition for the optimal price of the

Stackelberg follower is (Kadiyali/Sudhir/Rao (2001); Klapper (2003)):

$$(18) \quad \frac{\partial \pi_2}{\partial p_2} = (p_2 - c_2) \cdot b_{22} + q_2 = 0.$$

Inserting Equation (6) into Equation (18) and rearranging leads to:

$$(19) \quad p_2 = \frac{b_{22} \cdot c_2 - a_2 - b_{21} \cdot p_1}{2 \cdot b_{22}}.$$

The derivative of Equation (19), which is the derivative of the first-order condition of the profit of firm 2 with respect to p_1 , produces:

$$(20) \quad \frac{\partial p_2}{\partial p_1} = \frac{-b_{21}}{2 \cdot b_{22}}.$$

Thus, the first-order condition for the Stackelberg leader (firm 1) is:

$$(21) \quad \frac{\partial \pi_1}{\partial p_1} = (p_1 - c_1) \cdot \left(\frac{\partial q_1}{\partial p_1} + \frac{\partial q_1}{\partial p_2} \cdot \frac{\partial p_2}{\partial p_1} \right) + q_1 = 0.$$

Substituting Equations (5) and (20) into Equation (21) reveals:

$$(22) \quad (p_1 - c_1) \cdot \left(b_{11} - b_{12} \cdot \frac{b_{21}}{2 \cdot b_{22}} \right) + q_1 = 0.$$

Next, by using Equations (5) and (19) in Equation (22), I obtain:

$$(23) \quad p_1 = \frac{c_1 \cdot \left(b_{11} - \frac{b_{12} \cdot b_{21}}{2 \cdot b_{22}} \right) - a_1 - b_{12} \cdot p_2}{2 \cdot b_{11} - \frac{b_{12} \cdot b_{21}}{2 \cdot b_{22}}}.$$

It is noteworthy that the optimal prices that are described in Equations (19) and (23) can only be derived numerically. For this example, these optimal prices are $p_1 = 109.65\text{€}$ and $p_2 = 108.83\text{€}$ which yields profits of $\Pi_1 = 13,129,970\text{€}$ and $\Pi_2 = 13,347,237\text{€}$. Compared with the Nash-Bertrand solution, both prices and profits are slightly higher. Thereby, the profit of firm 2 (i.e., the follower) increases more strongly than the profit of firm 1 (i.e., the leader).

5.3 Collusive Pricing

In a collusive equilibrium, both firms choose their prices to maximize their combined profits. Therefore, each firm chooses prices for its products as if it were a monopoly firm that sold two differentiated products. The firms' objective in the collusive game (Kadiyali/Sudhir/Rao (2001)) corresponds to the profit function of the Niehans Rule (Niehans (1956); Skiera/Spann (1998)), as follows:

$$(24) \quad \pi = \pi_1 + \pi_2 = (p_1 - c_1) \cdot q_1 - C_{fix,1} + (p_2 - c_2) \cdot q_2 - C_{fix,2}.$$

Equation (25) then indicates the first-order condition for this collusive behavior:

$$(25) \quad \frac{\partial \pi}{\partial p_i} = (p_i - c_i) \cdot b_{ii} + (p_j - c_j) \cdot b_{ij} + q_i = 0 \quad (i = 1, 2),$$

and solving it yields the following optimal prices:

$$(26) \quad p_i = \frac{-2 \cdot b_{ij} \cdot p_j + b_{ii} \cdot c_i + b_{ij} \cdot c_j - a_i}{2 \cdot b_{ii}} \quad (i = 1, 2).$$

As in the Stackelberg leader-follower price competition, the optimal prices can only be obtained numerically. In this example, these optimal (identical) prices are $p_1 = 117.93\text{€}$ and

$p_2 = 117.93\text{€}$ which yields (identical) profits of $\Pi_1 = 14,238,264\text{€}$ and $\Pi_2 = 14,238,264\text{€}$. In line with logical expectations, the sum of the profits of both firms is higher in the collusive game ($\Pi_1 + \Pi_2 = 28,476,528\text{€}$) than in the Nash-Bertrand solution ($\Pi_1 + \Pi_2 = 26,227,907\text{€}$) or Stackelberg leader-follower price competition ($\Pi_1 + \Pi_2 = 26,477,207\text{€}$).

6 Influence of the Functional Form of the Price Response Function on the Effects of Price Changes

Although structural models' ability to capture strategic behavior is certainly attractive, their results remain sensitive to the functional form of the price response function. To illustrate this sensitivity, I assume a multiplicative functional form of the price response function instead of the linear one described in Equation (1):

$$(27) \quad q = \alpha \cdot p^\beta,$$

where:

- q: quantity,
- p: price,
- a, b: parameters of the linear price response function, and
- α, β : parameters of the multiplicative price response function.

The price elasticity in this multiplicative price response is $\varepsilon^{\text{mult}} = \beta$ (for details of the characteristics of functional forms, see Lilien/Kotler/Moorthy (1992); Hruschka (1996); Hanssens/Parsons/Schultz (2001)). Hildebrandt/Klapper (2001) and Hildebrandt/Klapper (2007) also outline some other functional forms to capture competitive effects.

To capture the situation outlined in the textbook example in Table 1, sales must be 1 million units at a price of 100€ as is the case for:

$$(28) \quad q(p) = 10.000.000.000 \cdot p^{-2}.$$

The corresponding profit of a price of 110€ is $\Pi^{\text{mult}} = 11,322,314\text{€}$ which compares reasonably well with the profit of $\Pi^{\text{linear}} = 10,909,000\text{€}$ obtained in the linear sales response function.

Optimizing the respective profit function shows that the optimal price in the case of the multiplicative price response function is:

$$(29) \quad p = \frac{\varepsilon}{1 + \varepsilon} \cdot c.$$

Equation (29) represents the inverse elasticity rule (Tirole (1988)), for which the optimal price is $p = 120\text{€}$ which produces a profit of $\Pi^{\text{mult}} = 11,666,667\text{€}$ still reasonably close to the corresponding linear profit of $\Pi^{\text{linear}} = 11,022,734\text{€}$

According to Equation (29), a cost decrease of 10% also lowers the optimal price by 10%, such that the optimal price for such a cost decrease is 108€ and the corresponding profit is $16,296,296\text{€}$ (which compares favorably with the corresponding profit of $16,368,186\text{€}$ for the linear price response function in Table 3).

Yet the results for multiplicative response functions differ substantively from those of the linear response functions if I take competition into consideration. Equations (30) and (31) describe the multiplicative sales response functions that capture these competitive prices:

$$(30) \quad q_1 = \alpha_1 \cdot p_1^{\beta_{11}} \cdot p_2^{\beta_{12}}, \text{ and}$$

$$(31) \quad q_2 = \alpha_2 \cdot p_1^{\beta_{21}} \cdot p_2^{\beta_{22}}.$$

In the multiplicative response functions, the optimal prices do not depend on the competitive prices, which is in sharp contrast to the case of linear response functions. The derivation of the optimal prices with Bertrand-Nash pricing clearly outlines this characteristic:

$$(32) \quad \frac{\partial \pi_1}{\partial p_1} = (p_1 - c_1) \cdot \alpha_1 \cdot \beta_{11} \cdot p_1^{\beta_{11}-1} \cdot p_2^{\beta_{12}} + \alpha_1 \cdot p_1^{\beta_{11}} \cdot p_2^{\beta_{12}} = 0.$$

Rearranging Equation (32) yields the optimal price for the first firm:

$$(33) \quad p_1 = \frac{\beta_{11}}{\beta_{11} + 1} \cdot c_1 \quad .$$

and the corresponding optimal price for the second firm thus is:

$$(34) \quad p_2 = \frac{\beta_{22}}{\beta_{22} + 1} \cdot c_2 \quad .$$

As Equations (33) and (34) show, the optimal prices in a Bertrand-Nash situation do not depend on the competitor's prices (i.e., $\frac{\partial p_1}{\partial p_2} = \frac{\partial p_2}{\partial p_1} = 0$). Therefore, the optimal prices in Ber-

trand-Nash and Stackelberg leader-follower settings are the same, which represents a substantially different result than the one obtained from the linear response function. A justified question is whether multiplicative response functions are adequate for structural models, which intend to explicitly make the competitors' reaction part of the model specification. My point of view is that the evaluation of the adequateness of the response function is an empirical question. If multiplicative response functions fit the data best, then they might be most adequate. Yet, it might be likely that other response functions (e.g., the linear response function or the logit model) fit the data better than the multiplicative response function.

The results for the collusion of both firms revert to similarity with the linear sales response function though, because both firms charge higher prices. To outline this similarity, I assume two price response functions that lead, at prices of 10€ to sales of 1,000,000 units:

$$(35) \quad q_1(p_1, p_2) = 1.000.000.000 \cdot p_1^{-2} \cdot p_2^{0,5}, \text{ and}$$

$$(36) \quad q_2(p_1, p_2) = 1.000.000.000 \cdot p_2^{-2} \cdot p_1^{0,5}.$$

More generally, these price response functions can be described as:

$$(37) \quad q_i(p_i, p_j) = \alpha_i \cdot p_i^{\beta_{ii}} \cdot p_j^{\beta_{ij}} \quad (i = 1, 2).$$

Taking the derivatives of the corresponding profit functions produces the following optimal prices that again correspond to the Niehans Rule and can only be derived numerically (Niehans (1956); Skiera/Spann (1998)):

$$(38) \quad p_i = \frac{\beta_{ii}}{\beta_{ii} + 1} \cdot c_1 - (p_j - c_i) \cdot \frac{q_j \cdot \beta_{ji}}{\beta_{ii} + 1} = \frac{\beta_{ii}}{\beta_{ii} + 1} \cdot c_1 - (p_j - c_i) \cdot \frac{\alpha_j \cdot p_j^{\beta_{jj}} \cdot p_i^{\beta_{ji}}}{\beta_{ii} + 1} \cdot \frac{\alpha_i \cdot p_i^{\beta_{ii}} \cdot p_j^{\beta_{ij}}}{\beta_{ii} + 1} \cdot \beta_{ji}.$$

For this example, the optimal (identical) prices are $p_1 = p_2 = 180.00\text{€}$ which yields (identical) profits of $\Pi_1 = \Pi_2 = 49,690,399\text{€}$. As should be expected, the sum of the profits of both firms is greater in the collusive scenario ($\Pi_1 + \Pi_2 = 99,380,799\text{€}$) than for either the Nash-Bertrand or Stackelberg leader-follower price competition (in both these cases, $\Pi_1 + \Pi_2 = 86,227,907\text{€}$).

Table 5 summarizes the results for the linear and multiplicative price response functions, which both mimic a price elasticity of -2 and the same profit for the described "current situation". Although the prices in the Nash-Bertrand and Stackelberg leader-follower pricing scenarios differ for the linear price response function, the differences are rather small. The most remarkable difference reveals that the collusive prices are on average only 7.96% higher for the linear price response function but 50% higher for the multiplicative price response function. Still, both results indicate that firms have an incentive to move from a Nash-Bertrand Pricing competition to collusive pricing.

Table 5: Summary of results

	Current Situation	Optimal Prices (without competitor's reaction)	Nash-Bertrand Pricing	Stackelberg Leader-Follower Pricing	Collusive Pricing
Linear Price Response Function					
Price of firm 1	100,00€	107.50€	108.70€	109.65€	117.93€
Price of firm 2	100,00€	100.00€	108.70€	108.83€	117.93€
Quantity of firm 1	1,000,000	836,636	885,375	868,636	763,636
Quantity of firm 2	1,000,000	1,037,500	885,375	887,767	763,636
Profit of firm 1	10,000,000€	11,022,734€	13,113,953€	13,129,970€	14,238,264€
Profit of firm 2	10,000,000€	11,500,006€	13,113,953€	13,347,237€	14,238,264€
Total profit of both firms	20,000,000€	22,522,740€	26,227,907€	26,477,207€	28,476,528€
Multiplicative Price Response Function					
Price of firm 1	100,00€	120.00€	120.00€	120.00€	180.00€
Price of firm 2	100,00€	100.00€	120.00€	120.00€	180.00€
Quantity of firm 1	1,000,000	694,444	760,726	760,726	414,087
Quantity of firm 2	1,000,000	1,095,445	760,726	760,726	414,087
Profit of firm 1	10,000,000€	11,666,667€	15,643,547€	15,643,547€	19,690,399€
Profit of firm 2	10,000,000€	13,817,805€	15,643,547€	15,643,547€	19,690,399€
Total profit of both firms	20,000,000€	25,248,471€	31,287,093€	31,287,093€	39,380,382€
Notes: Variable costs per unit = 60€, fixed costs = 30,000,000€					

7 Summary

This article compares the ability of structural and reduced-form models to improve pricing decisions. Both models allow for capturing the effect of consumers' responses and changes in cost structure on prices. Yet, only structural models capture competitors' reactions to price adjustments and are thus able to describe market equilibriums. However, this ability comes at the cost of the requirement to make the assumption that firms behave optimal. In essence, this assumption means that firms cannot improve prices because they already set optimal prices. Consequently, structural modeling in its most basic form does not suggest ways to increase profits further by altering prices in an observed situation. A situation like the "current situation" in the example herein would never be observed in structural models, because the firms obviously do not behave optimally in this situation. Rather, depending on the competitive setting, the Nash-Bertrand, Stackelberg leader-follower, or collusive prices (or prices that result from other competitive settings) appear in a market so that researchers can observe which pricing behavior firms follow. Additionally, structural models provide clear implications for firms concerning the pricing strategies that firms should follow. Firms, for example, should try to avoid a Bertrand-Nash pricing strategy and aim at reaching collusive pricing because it increases total profit (although not necessarily welfare). Additionally, structural models also enable to improve pricing decisions if environmental conditions change. Chintagunta/Dubé/Singh (2003), for example, determine optimal prices under different opportunities to discriminate prices. In particular, they show that a store-pricing policy instead of a zone-pricing policy, i.e. a policy under which prices differ between all stores instead of a policy under which similar prices are charged for all stores of the same regional zone, generates substantial incremental profit for the retailer.

The assumption of optimal behavior of firms contrasts sharply with that depicted by reduced-form models, which do not assume that all firms behave optimally and therefore can make suggestions about improving prices. Yet reduced-form models also are limited in their

insights into the new equilibriums that arise when factors, such as the costs of suppliers, change. As such, both models have their unique characteristics and offer promise for different areas of application.

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